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DOI: https://doi.org/10.1080/10255842.2020.1724975

Originally published at:
Mehl, Albert (2020). Is it possible to detect a true rotation axis of the temporomandibular joint with common pantographic methods? A fundamental kinematic analysis. Computer Methods in Biomechanics and Biomedical Engineering, 23(9):445-455.
DOI: https://doi.org/10.1080/10255842.2020.1724975

Is it possible to detect a true Rotation Axis of the Temporomandibular Joint with common Pantographic Methods? A fundamental kinematic Analysis.

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Abstract

The location of the terminal hinge axis of the temporomandibular joint is still a very wide-spread procedure in dentistry in order to replicate the movement in various articulator devices. Especially pantographic methods are claimed to provide accurate measurements and, additionally, are seen to be able to separate a pure rotation of the joint from a movement with an arbitrary combined shift and rotation. In the latter application, these methods were used in a lot of studies as a reference standard. The aim of this study was to analyze, whether common pantographic methods in general are able to distinguish between a pure rotation and a movement with rotational and translational portions. The mathematical proof of this analysis was done with theoretical kinematic considerations and compared with computer simulations. The results show for the first time that there exist combinations of rotational and translational movements of the temporomandibular joint which cannot be separated from pure rotational movements using actual pantographic methods. Even more, the consequence is a shifted location of the (combined) finite center (axis) of rotation in comparison to the true center (axis) of rotation: in case of a translational portion of only 1 mm, this is a displacement of around +/- 6 mm and, in case of 2 mm translation, a displacement of +/-12 mm. This finding necessitates a critical reinterpretation of former studies using pantographic methods as a reference standard. Further, under some circumstances it may also affect the applicability of articulator concepts and the interpretation of functional signs.

Introduction

Recording and analyzing individual mandibular movement are seen to be important both for diagnostic purposes and for restorative treatments. Different approaches have been established for these tasks in dentistry. The most widespread approach is the replication of the mandibular movement in a physical articulating system, using stone replicas of the individual dentition. Such physical articulator systems are designed to be adjustable to some specified individual parameters like the position of the rotation axis, in dentistry also called the terminal or transversal hinge axis (THA), the sagittal and horizontal inclination of the movement of this axis, and some additional characteristic joint parameters.

The determination or knowledge of the position of the THA plays a central role for this articulator concept. It is assumed that the mandible performs a movement, which can be described via a combination of rotations and translations of this axis (Catic and Naeije 1999). Following that assumption, the clinical processes first imply the determination of the axis of rotation (e.g. individual and arbitrary terminal hinge axis) followed by the determination of angles and translational shifts of some specified axis movements (sagittal inclination, Bennett angle, immediate side shift etc.) (Ahlers et al. 2015). With these values, the physical articulators can be programmed to approximate the mandibular movement with only a few parameters.

A very common method for the location of the true individual hinge axis, especially used as a reference standard in dentistry, was first described by McCollum (1939 and 1960) and then successively improved upon in later decades (Lauritzen 1961; Bosman 1974). The basic principle of this method is that a pin fixed to a device, which is connected to the mandibular teeth, traces the movements on a frame, which itself is fixed to the head as a reference system (Fig. 1 left) (Lauritzen 1974). In case of a rotational movement, the pin traces exact circular segments whose sizes depend on the distance to the center of rotation (Fig. 1 right). The closer the position of the pin to the center of rotation, the shorter is the circular segment. The hinge axis can be determined by iteratively changing the position of the pin until the point is detected, where a movement can no longer be observed (Slavicek 1988). This method is also known as pantographic kinematic method (Bosman 1974). Further implementations of this method may use electronic or optical tracking devices, including the tracing of the circular segments and indirectly inferring the center of rotation (Slavicek 1988; Piehslinger et al. 1991).

Besides the location of the hinge axis, these pantographic methods can be used in a further application: in case of a pure rotation there exists a point (center of rotation), which will not change the position during movement, whereas in case of an additional translation no such point can be found. The same is true for detecting possible deviations from an exact circular trace segment. Under these aspects it should be possible to differentiate between a movement with pure rotation or with combined rotation and translation (Bosman 1974; Lückenbach and Eisenmann 1991).

Up to now, these properties have been applied for a long time in therapeutic rehabilitations, and the pantographic method has even been served as a reference in a lot of studies. Such studies draw conclusions using the above mentioned properties, e.g. for the position of the true hinge axis by excluding additional translational movements (Lauritzen 1974; Slavicek 1988; Hugger et al. 2001; Bias and Kordass 2009), for the location of arbitrary condylar points in relation to the individual kinematic points (Nagy et al. 2002), about the influence of using arbitrary points in comparison to individual hinge axis points on the sagittal and transversal inclination angles (Bernhardt et al. 2003), about the errors onto the occlusion using arbitrary points instead of individual points for articulator programming (Morneburg and Pröschel 2002; 2011) and about the physiological condyle position in relation to the fossa (Stamm et al. 2004). However, a new investigation points out in an incidental finding that in some joint movement situations the above mentioned properties of the pantographic method do not hold and, therefore, conclusions from the results of the above mentioned studies can only be drawn with the utmost caution (Mehl 2018).

According to those contradictions, it is essential to analyze the fundamental properties and constraints of the hinge axis concept (axis of rotation) in more detail. Therefore, the aim of this study was, first, an analysis, which should provide answers to the problem whether a pure rotational movement can be separated from a combined translational and rotational movement using pantographic methods. In a second step, some effects onto the location of the terminal hinge axis or axis of rotation were investigated numerically. The entire analysis was performed in the scope of a mathematical proof applying theoretical kinematic considerations and was justified by special computer simulations.

Material and Methods

Kinematic of Mandibular Movement

As far as possible, the terminology and standardized representation of kinematic data were used as recommended by the Glossary of Prosthodontic Terms (Ferro et al. 2017) and the International Society of Biomechanics (Wu and Cavanagh 1995). The *global reference coordinate* system is fixed to the head or the maxilla, respectively (Fig. 2). The moving or *local coordinate system* is connected with the mandible. Keeping the focus on the aim of the study, it can be assumed, without loss of generality, that all relevant jaw movements are planar movements and are described in the sagittal plane, whenever it simplifies the calculation (Fig. 2).

The movement of the mandible was analyzed with the help of fundamental kinematic equations. Special focus was put on the properties of uniform motion, i.e. a motion of constant translational and angular velocity. For this purpose, general solutions for the differential equations were developed and the behavior of certain reference points was calculated. The result section lists the detailed mathematical derivation and the corresponding terminology.

Application of kinematic aspects to the temporomandibular joint

In case of hinge axis determination for the temporomandibular joint, the relevant range of mouth opening is seen at values of 7 to 15 mm (Hugger and Kordass 2017; Okeson, 2019). For calculations in this study, a 15 mm opening was assumed. The reason is that any effects, which may differentiate the pure rotation from a combination of rotation and translation, will be more pronounced with 15 mm rather than with smaller opening values like 10 mm or 7 mm. Therefore, if it is not possible to detect differences for a 15 mm opening movement, they will even be less detectable for smaller opening values. The radius of opening movement for the incisal point (between the two lower first incisors) can be calculated from the average Bonwill triangle (with arms of 103.5 mm, a base of 99.6 mm, and a height from occlusal plane to condyle center of 35 mm) (Maggetti et al. 2015) and equals 90.6 mm. With this radius and 15-mm opening, the maximum relevant angle of opening corresponds to 9.6° degrees. The radius of the condyle was set to 4 mm (Peroz et al., 2011). These parameters are the basis for further calculations including the MATLAB simulations (see also Fig. 3).

Computer Simulation

In order to justify the analytic considerations described above, a simulation was performed (analog to Mehl 2018). A schematic model of a mandibular body was designed in MATLAB (Version R2018a, MathWorks, USA) (Fig. 3). The same geometric parameters as described before were used: radius of opening 90.6 mm (distance from center of condyle to incisal point), height from occlusal plane to condyle center 35 mm, mouth opening 15 mm, maximum opening angle 9.6°, radius of condyle 4 mm. These average values used here for the geometric model can also be interpreted as representations of individual values taken from real patients: even limited to the arbitrary database of 120 individuals (used in Maggetti et al. 1995), 3 real individuals can be found from this database, who have a difference of less than 1 mm in all parameters compared to the average values (a more precise measurement than 1 mm of this parameters is hardly to achieve, even in CTs). The rotations of the lower jaw could be performed around the center of the condyle by incremental steps of small angles until the entire rotation was reached. Additionally, the translation could also be applied incrementally and simultaneously with each incremental rotation, so that all possible (approximately continuous) combinations of rotational and translational movements could be constructed (Fig. 3 and 4). The number of incremental steps were chosen with n=30. Traces of sample points around the finite center of rotation (FCR, corresponds approximately also to the point P_{Min}) were recorded and presented in the respective figures. The maximum amplitude (maximum length of the segments) of the respective traces were also calculated and plotted into the figures. Additionally, the MATLAB code for this simulation is made available in the supplementary material.

Results

Derivation of the kinematic formulas

i) <u>Basic Kinematics</u>

In general, the movement of an arbitrary point *P* on a rigid body (or connected with the rigid body) can be described by a translation of a reference point *O* and a rotation around this reference point *O* (rotation axis through this point *O* with angular velocity $\vec{\omega} = (\omega_1, \omega_2, \omega_3)$). The velocity of the point *P* ($\dot{\vec{R}}_P$) is then given by:

$$\dot{\vec{R}}_P = \dot{\vec{R}}_O + \vec{\omega} \times \left(\vec{R}_P - \vec{R}_O\right) = \dot{\vec{R}}_O + \vec{\omega} \times \vec{r}_P \ ; \qquad (1)$$

It can be shown, that $\dot{\vec{R}}_P$ is independent on the reference point *O* ($\vec{R}_O$), i.e., for another arbitrary reference point O' the following equation describes the same movement of the Point *P*:

$$\dot{\vec{R}}_P = \dot{\vec{R}}_{O'} + \vec{\omega} \times \left(\vec{R}_P - \vec{R}_{O'}\right) = \dot{\vec{R}}_{O'} + \vec{\omega} \times \vec{r'}_P = \dot{\vec{R}}_O + \vec{\omega} \times \vec{r}_P \qquad (2)$$

It is important to mention that these relations are time dependent, as all variables are a function of time *t* and all position vectors $\vec{R}_P$, $\vec{R}_O$ and $\vec{R}_{O'}$ are referenced to a fixed global reference coordinate system. In order to obtain the position or path of *P*, an integration over time has to be made, i.e. $\vec{R}_P(t_s) = \int_{t_0}^{t_s} \dot{\vec{R}}_P(t)dt$, and in order to get the rotation angles, in an analog manner $\vec{\alpha}(t_s) = \int_{t_0}^{t_s} \vec{\omega}(t)dt$ (in the planar case the last relation without vectors, only scalars).

With regard to a fixed global coordinate system, from these fundamental relations a few important conclusions can be drawn:

a) For each arbitrary point *P* on the mandible, the angular velocity $\vec{\omega}$ is the same, independently of the reference point used for the rotation axis of the rigid body.
b) According to a), $\vec{\omega}(t)$ is always the same for each time point *t*, therefore the rotation angle (in 3D the three angles) by integration is the same for each point on the mandible, independently of the reference point *O* defined.
c) With the same arguments, the path of any point P on the mandible is independent on which reference point or rotation axis position *O* is chosen.

In the planar case $\vec{\omega}$ reduces to $\vec{\omega} = (0,0,\omega)$ and equation (1) can be rewritten as follows:

$$\dot{X}_P = \dot{X}_O - \omega \cdot (Y_P - Y_O) = \dot{X}_O - \omega \cdot y_P \;; \tag{3}$$

$$\dot{Y}_P = \dot{Y}_O + \omega \cdot (X_P - X_O) = \dot{Y}_O + \omega \cdot x_P \;; \tag{4}$$

with $\vec{R}_P = (X_P, Y_P)$, $\vec{R}_O = (X_O, Y_O)$, and $\vec{r}_p = (x_P, y_P) = (X_P - X_O, Y_P - Y_O)$.

A further important measure is the path length or arc length, which is passed by a point P on a rigid body. This path length is given by the velocity of the point *P* ($\dot{\vec{R}}_P$) according to following relation:

$$s(t_s) = \int_{t_0}^{t_s} \left\| \dot{\vec{R}}_P(t) \right\| dt \tag{5}$$

In a planar case, equation (5) can be rewritten to:

$$s(t_s) = \int_{t_0}^{t_s} \sqrt{\dot{X}_P^{\;2}(t) + \dot{Y}_P^{\;2}(t)}\, dt \tag{6}$$

ii) <u>Instantaneous Center of Rotation (ICR) or Finite Center of Rotation (FCR)</u>

According to the statements before, one is free to decide where to position the origin around which the rotation occurs (= reference point *O*). One point of special interest is where no translational movement occurs: this point on the mandible stays fixed and the whole mandible performs a pure rotation around this point. For each time value t, this point CR can be found by setting the velocity $\dot{\vec{R}}_{CR} = 0$, therefore $\vec{R}_{CR}$ can be calculated from:

$$\dot{\vec{R}}_{CR} = 0 = \dot{\vec{R}}_O + \vec{\omega} \times \left(\vec{R}_{CR} - \vec{R}_O\right);$$

$$\vec{\omega} \times \vec{R}_{CR} = \vec{\omega} \times \vec{R}_O - \dot{\vec{R}}_O; \tag{7}$$

In the planar case, point CR ($\vec{R}_{CR} = (X_{CR}, Y_{CR})$ can be calculated from:

$$X_{CR} = X_O - \frac{\dot{Y}_O}{\omega} \;; \tag{8}$$

$$Y_{CR} = Y_O + \frac{\dot{X}_O}{\omega} \;; \tag{9}$$

The solution $\vec{R}_{CR} = (X_{CR}, Y_{CR})$ is unique as long as $\omega \neq 0$. For each time value *t*, a unique center of rotation exits, which, of course, may change from time to time. This time-dependent center of rotation is therefore also called as *instantaneous center of rotation* (ICR) and it is an “infinitesimal” measure due to the first deviation. In case of $\omega = 0$, there is a pure translation and the ICR lies by definition in the infinite.

In practice, 1.) a measurement of $\vec{R}_O$ and $\vec{\omega}$ does not deliver continuous values, but rather discrete values of positions according to the discrete time steps, and 2) $\vec{\omega}$ can only be determined indirectly. Therefore, the continuous relation (7) has to be approximated by discrete differences. In such cases, another approach is more appropriate to calculate $\vec{R}_{CR}$. Each rigid transformation of an object in space between *two* positions (e.g. at times t_i and t_{i+1}) can be described as

$$\vec{R}_P(t_{i+1}) = \mathbf{R} * \vec{R}_P(t_i) + \vec{s} \tag{10}$$

where $\mathbf{R}$ is the rotation matrix (defining the rotation axis direction $\vec{n}$ and the rotation angle φ) and

$\vec{s}$ the shift or translation vector. Then, for each arbitrary transformation according to (10) a coordinate system with a new origin *O* can be found in *2D*, where $\vec{R}_P(t_{i+1}) - \vec{R}_O$ and $\vec{R}_P(t_i) - \vec{R}_O$ performs a pure rotation, i.e. $\vec{s} = 0$, or in *3D* a rotation around the axis $\vec{n}$ and a translation $\vec{s}$ along this axis, i.e. $\vec{s} \parallel \vec{n}$. In a clinical or experimental setup, in general two or more marker positions $\vec{R}_{P_k}$, with $k = 2, \dots, m$, at times t_i and t_{i+1} are measured. The task is then to find the best $\mathbf{R}$ and $\vec{s}$ under the assumption that e.g. the measurement errors are normal distributed. Then, the quadratic error between calculated and measured coordinates should be minimal:

$$\sum_{k=1}^{m} \left[\vec{R}_{P_k}(t_{i+1}) - \mathbf{R} * \vec{R}_{P_k}(t_i) - \vec{s}\right]^2 = min$$

The best $\mathbf{R}$ and $\vec{s}$ can be determined by mathematical optimization, applying Singular Value Decomposition or Eigenvalue methods. In case of a pure rotation, the reference point *O* or equivalently the origin *O* of a new reference coordinate system lies on the rotation axis, i.e. the translation $\vec{s}$ in this coordinate system vanishes. In planar 2D case, this condition allows the calculation of a point $\vec{R}_{CR'}$ as the rotation center by

$$\vec{R}_{CR'} = (\mathbf{R} - \mathbf{I})^{-1} * \vec{s};$$

with $\mathbf{I}$ as the identity matrix. In case of a rotation angle $\varphi \neq 0$, i.e. $\mathbf{R} \neq \mathbf{I}$, this solution is unique for specified finite time interval t_i and t_{i+1}. $\vec{R}_{CR'}$ is called here the *finite center of rotation* (FCR).

From these relations following conclusions can easily be drawn:

a) The FCR $\vec{R}_{CR'}$ in general is strongly dependent on the chosen time interval t_i and t_{i+1}
b) The larger the time step $\Delta t = t_{i+1} - t_i$, the larger in general the deviation between the ICR $\vec{R}_{CR}$ and the FCR $\vec{R}_{CR'}$.
c) From a) and b) in particular follows, that the ICR $\vec{R}_{CR}$ and the FCR $\vec{R}_{CR'}$ in general do not coincide.

Calculating $\vec{R}_{CR}(t)$ at each time point t results in a continuous path describing the movement of the ICR. The path of the ICR is also defined as the *centrode*. The tracking of the FCRs of subsequent time steps results in the discrete version of the centrode (so called *path of the FCR*). Again, it is important to keep in mind that the ICR path may differ from the FCR path depending on the kind of kinematic movement or the time steps and intervals chosen. In literature, FCR is mostly also named as ICR due to the fact that real measurements can only record values at discrete time steps. Further, according to the time steps (or interval positions) used, the positions or paths of the FCR will vary. This should also be kept in mind, when comparing and interpreting the results of different studies.

iii) Pure rotation versus combination of rotation and translation – theoretical aspects

With an observation referenced to a global coordinate system, it is theoretically possible to detect if a rigid body performs a pure rotation around a fixed axis or a combined rotational and translational movement with moving axes. This possibility is based on the fundamental relationship, which can be derived from the above relations and which states that a pure rotation is only present if the path of the ICR collapses to a point or, equivalently, for each time t the ICR is the same and does not change: $\vec{R}_{CR}(t) = \vec{R}_{CR}(t_0) = const., \forall t \in [t_0, t_{end}]$. Further, it can also be derived from this statement, that for any two discrete time step intervals $\Delta t_{j,v} = t_{j+v} - t_j$ and $\Delta t_{k,w} = t_{k+w} - t_k$ with

$t_j, t_{j+v}, t_k, t_{k+w} \in [t_0, t_1, \dots, t_n]$, the FCR with $\vec{R}_{CR'_{j,v}}$ and the FCR with $\vec{R}_{CR'_{k,w}}$ must coincide. Additionally, it follows that FCR and ICR are the same and only the same in case of a pure rotation.

These fundamental relationships therefore provide strategies to detect, if a pure rotation of the rigid body is present:

a) Determination of the ICR over time: if the ICR does not move and stays stable on a fixed point then a pure rotation is present.
b) Measuring two or more FCRs at different time steps: if one of these FCRs does not fall on the same point, then no pure rotation is present and the movement is a combination of a rotation and a translation.

And following a) and b):

c) Searching directly for a point fixed to the mandible which does not move during the entire motion: if such a point can be found this point represents the axis for a pure rotational movement.

The possibilities a) and b) can be considered as *FCR (or ICR) based methods* and c) as *trace tracking methods*.

iv) Uniform Rotation and Translation (constant angular and translational velocity)

In order to discuss some of the above deduced relations in a planar case in more detail, a combined movement of a uniform rotation and translation is assumed as a representative example. This means that the rigid body moves with both a *constant* angular and translational velocity (ω, $\dot{X}_O$ and $\dot{Y}_O$ are constant). Without loss of generality, the reference point *O* can be chosen as the point, around which the rotation is performed and simultaneously the translation takes place with following properties: during the time interval T, the rotation around the point *O* covers the angle $\Delta\alpha$ and the point *O* is shifted by Δx and Δy:

$$\omega = \frac{\Delta\alpha}{\mathrm{T}}\ ;\ \dot{X}_O = v_x = \frac{\Delta x}{\mathrm{T}}\ ;\ \dot{Y}_O = v_y = \frac{\Delta y}{\mathrm{T}}\ ; \tag{11}$$

Additionally, without loss of generality the time interval can be scaled from $[0, T]$ to $t \in [0,1]$ with $T = 1$.

Using these uniform movements, for any arbitrary point *P* the time dependent relations (3) and (4) can be solved analytically (solution of a second order linear inhomogeneous differential equation with a first grade polynomic perturbation function) resulting in:

$$X_P(t) = r_{x_0}\cos\omega t - r_{y_0}\sin\omega t + v_x t + x_O; \tag{12}$$

$$Y_P(t) = r_{y_0}\cos\omega t + r_{x_0}\sin\omega t + v_y t + y_O; \tag{13}$$

with

$$\begin{pmatrix} r_{x_0} \\ r_{y_0} \end{pmatrix} = \begin{pmatrix} X_P(t=0) - X_O(t=0) \\ Y_P(t=0) - Y_O(t=0) \end{pmatrix} = \begin{pmatrix} X_P(t=0) - x_O \\ Y_P(t=0) - y_O \end{pmatrix}$$

the respective distances and point coordinates at start time $t = 0$.

One important point of interest is the ICR $\vec{R}_{CR}$ at time t and the path of the ICR during the entire movement from $t = 0$ to T. For the position of the ICR $\vec{R}_{CR}$ at time t the following conditions must hold (see also (7)): $\dot{X}_P(t) = 0$ and $\dot{Y}_P(t) = 0$.

Applying this to equations (12) and (13) and solving for the unknowns r_{x_0} and r_{y_0}, following relation is obtained:

$$r_{x_0} = \frac{v_x\cos\omega t + v_y\sin\omega t}{\omega}; \qquad r_{y_0} = \frac{v_x\sin\omega t - v_y\cos\omega t}{\omega};$$

As r_{x_0} and r_{y_0} is the start position relative to the reference point *O*, the position of the ICR at time t and therefore the path of the ICR or centrode can be calculated by resubstituting r_{x_0} and r_{y_0} into (12) and (13):

$$X_{CR}(t) = v_x t - \frac{v_y}{\omega} + x_O;$$

$$Y_{CR}(t) = v_y t + \frac{v_x}{\omega} + y_O;$$

or with the constant values defined in (11) and scaling the time interval from $[0, T]$ to $t \in [0,1]$:

$$X_{CR}(t) = \Delta x \cdot t - \frac{\Delta y}{\Delta\alpha} + x_O; \tag{14}$$

$$Y_{CR}(t) = \Delta y \cdot t + \frac{\Delta x}{\Delta \alpha} + y_O; \tag{15}$$

The relations (14) and (15) show that the ICR moves on a linear path with totally Δx in x-direction and Δy in y-direction. At start position the ICR is located with a difference vector of $(-\frac{\Delta y}{\Delta \alpha}, \frac{\Delta x}{\Delta \alpha})$ relative to the reference point O.

Another important point of interest is, where the minimum path length during movement (see equation 6) is obtained (P_{Min}). In general, this can only be solved numerically via an elliptic integral. However, a good approximation can be made by assuming that the start point and the end point of the movement coincides, i.e. $X_P(t = T) - X_P(t = 0) = 0$ and $Y_P(t = T) - Y_P(t = 0) = 0$. Applying this condition to (12) and (13) and using (11), following result for a candidate P_{Min} of a very small path length can be obtained:

$$r_{x_0} = \frac{\Delta x}{2} - \frac{\Delta y}{2 \cdot \tan\frac{\Delta \alpha}{2}}; \quad r_{y_0} = \frac{\Delta y}{2} + \frac{\Delta x}{2 \cdot \tan\frac{\Delta \alpha}{2}}; \tag{16}$$

For small angles $\Delta \alpha < 0.26\ (15°)$, the equation (16) can be further reduced to

$$r_{x_0} = \frac{\Delta x}{2} - \frac{\Delta y}{\Delta \alpha}; \quad r_{y_0} = \frac{\Delta y}{2} + \frac{\Delta x}{\Delta \alpha}; \tag{17}$$

The maximum amplitude of this special point will approximately lie at a position at time $t = \frac{T}{2}$, i.e. the distance from start and end point can be calculated from $\Delta s_x = X_P\left(t = \frac{T}{2}\right) - X_P(t = 0)$ and $\Delta s_y = Y_P\left(t = \frac{T}{2}\right) - Y_P(t = 0)$. With (16) inserted in (12) and (13) and using parameters from (11), this results in an amplitude of:

$$\Delta s_x = \frac{\Delta y}{2} \cdot \tan\frac{\Delta \alpha}{4}; \quad \Delta s_y = \frac{\Delta x}{2} \cdot \tan\frac{\Delta \alpha}{4}; \tag{18}$$

To complete the parameters describing the uniform movement, the FCR $\vec{R}_{CR'}$ should also be deduced here. According to equation (10) and the following discussion there, the FCR is given by the approximation of the velocity measured between time t and $t + \Delta t$, where the velocity vanishes, i.e. $\dot{\vec{R}}_{CR'}(t + \Delta t) = \frac{\vec{R}_{CR'}(t+\Delta t) - \vec{R}_{CR'}(t)}{\Delta t} = 0;$

Applying the equivalent condition $\vec{R}_{CR'}(t+\Delta t)-\vec{R}_{CR'}(t)=0$ to equations (12) and (13), solving for $r_{x_0}$and r_{y_0} and resubstituting into (12) and (13) at $\vec{R}_{CR'}(t)$, the following exact relation for the FCR between time t and $t+\Delta t$ is obtained:

$$X_{CR'}(t)=v_x(t+\tfrac{\Delta t}{2})-v_y\frac{\Delta t}{2\tan(\frac{1}{2}\omega\Delta t)}+x_O; \tag{19}$$

$$Y_{CR'}(t)=v_y(t+\tfrac{\Delta t}{2})+v_x\frac{\Delta t}{2\tan(\frac{1}{2}\omega\Delta t)}+y_O; \tag{20}$$

With the values defined in (11), scaling the time interval from $[0,T]$ to $t,\Delta t\in[0,1]$, and for small angles $\Delta\alpha<0.26\ (15°)$, the equations can be further reduced to

$$X_{CR'}(t)=\Delta x\cdot(t+\tfrac{\Delta t}{2})-\frac{\Delta y}{\Delta\alpha}+x_O; \tag{21}$$

$$Y_{CR'}(t)=\Delta y\cdot(t+\tfrac{\Delta t}{2})+\frac{\Delta x}{\Delta\alpha}+y_O; \tag{22}$$

which shows no differences to the position of the ICR from (14) and (15) at time $t+\frac{\Delta t}{2}$. However, whereas (14) and (15) are exact solutions, the exact solution for the FCR is (19) and (20). If the tan-function is additionally approximated by taking into account the third polynomic grade, the slight differences between ICR and FCR can be represented by $\pm\Delta y\cdot\frac{\Delta\alpha\cdot\Delta t^2}{24}$ or $\pm\Delta x\cdot\frac{\Delta\alpha\cdot\Delta t^2}{24}$, respectively. If $\Delta t\to 0$, then FCR converges to ICR.

Interpretation of the Kinematic Calculations

Important results from the above derivation are the formulas for the instantaneous center of rotation (ICR) with (14), (15), for the finite center of rotation (FCR) with (21), (22) and for the maximum amplitude of the path with (18), to name a few. Based on these theoretical considerations of a uniform movement, some relevant feature points and distances are presented in Figure 3. The rotation is assumed around a reference point O with a constant angular velocity $\Delta\alpha/T$, simultaneously shifting this point O with constant velocities $\Delta x/T$ and $\Delta y/T$ (see equation (11)). This corresponds to a typical linear hinge axis movement. The point with approximately the least movement will lie at the position r_{x_0} and r_{y_0} relative to the reference point O with the respective distance d_{axis}. According to (18), the maximum length of the amplitude of the path at that point P_{Min} will be approximately given by

$$\Delta l_{min} = \sqrt{\Delta s_x^2 + \Delta s_y^2} = \frac{1}{2} tan\frac{\Delta\alpha}{4} \cdot \sqrt{\Delta x^2 + \Delta y^2}\ ;$$

Table 1 shows some representative values for different temporomandibular joint movements using the notations and measures from Fig. 2 and 3. For ease of discussion and without loss of generality, the orientation of the coordinate system was chosen to have the x-axis orientated along the direction of the linear movement, i.e. $\Delta y = 0$. In a general case with $\Delta y \neq 0$, the length $\sqrt{\Delta x^2 + \Delta y^2}$ corresponds to Δx in Tab. 1, whereas the other values d_{axis} and Δl_{min} do not change (the position P_{Min} with r_{x_0} and r_{y_0} is only rotated according to Δx and Δy).

The values in Tab. 1 give some remarkable insights into the kinematics of the temporomandibular joint. As an example, in case of a mouth opening with 9.6° and a translation of the condyles with 1 mm, the point P_{Min} with the least path length will lie at a distance of 6 mm from the true rotation axis (=reference point O). If it is *not* known, in which direction the translation of 1 mm will occur, the point P_{Min} can lie anywhere on a circle around O with a radius of 6 mm, i.e. with an uncertainty of +/- 6 mm. Interestingly, the detectable path amplitude Δl_{min} at that point P_{Min} is around 21 µm. Other combinations show, that e.g. in case of mouth opening of 9.6° and translation of 2 mm or 6.4° and 1 mm respectively, P_{Min} is located at +/- 12 mm or +/- 9 mm, respectively and Δl_{min} is 42 µm or 14 µm, respectively.

Computer Simulation

The results of the MATLAB simulation confirm the results of the values d_{axis} and Δl_{min} from the equations (17) and (18) and table 1, respectively. Fig. 1 (right) displays an example of a pure rotational movement around the condyle center and its corresponding trace recordings. In Fig. 4 the results of simulations are shown for a movement, where the condyle performs a combined translation and rotational movement (4a with 1.1 mm and 4c with 2.2 mm). The magnification of the respective traces around the finite center of rotation (FCR) is presented in Figs. 4b and 4d, respectively. It can be clearly demonstrated that the pattern of traces does not visibly change, even if the amount of translation increases. Furthermore, the location of the point P_{Min} changes dramatically even for small values of translational movements.

Discussion

The question, if it is possible to differentiate between a pure rotational or a combined translational and rotational movement of the mandible using pantographic methods, has been raised. Theoretical kinematic considerations and computer simulations were conducted. The results clearly demonstrate that there exist combinations of rotational and translational movements, where the translational part cannot be detected with *current* devices. Furthermore, in such cases the "true" center of rotation (true THA) and the detected center of rotation (corresponding to the FCR) are located at different positions. The distance between these positions is highly dependent on the portion of the translational path.

The true terminal hinge axis in dental functional diagnostic is defined as an imaginary transverse axis between two *fixed* points (or points at rest), around which a *pure* rotational movement of the mandible exists (Bias and Kordass 2009). It is also well known, that in general the hinge axis does not coincide with the center of the condyle or the intercondylar axis (Hugger et al. 2001; Bias and Kordass 2009; Morneburg and Pröschel 2011). In dentistry different systems and processes are used for the determination of this rotation axis. As described above a very common and often used method, especially as a golden standard in many investigations, is based on trace tracking (McCollum 1963; Lauritzen 1974). The theoretical background of these pantographic methods was interpreted in a way that it should be possible to differentiate between a movement with pure rotation or with combined rotation and translation: the true hinge axis can be detected or it can at least be shown that an additional translation is present. However, these considerations do not include possible errors introduced by the measurements with such methods and by the limits of the possible accuracies of such systems. According to Tab 1, this pantographic method must be able to resolve e.g. at least 42/2 µm $\approx$ 21 µm in order to safely detect an additional translational proportion of 2 mm. With only visible detection of the traces and the pin, this clearly seems impossible. Even for electronical and optical realizations of this method it would actually be highly unlikely that this is realistic. Values from manufacturers and some rare investigations in literature report accuracies of around 100 to 200 µm (Bias and Kordass 2009; Hugger et al. 2001; Gamma 2018; Franz et al. 2014; Sicat 2016; Kavo 2019).

The new finding of this study is that the mentioned methods are not able to distinguish between a pure rotation and a translation and rotation. As a consequence, not only rotational but also translational parts may be included in such “true” hinge axis measurements despite the studies describing that e.g. patients with translational movements were excluded (see also Hugger and Kordass 2017, Okeson 2019). This could explain the relatively high variation and discrepancy of investigated hinge axis points from various studies (Hugger et al. 2001; Morneburg and Pröschel 2011; Hugger and Kordass 2017). Additionally, this important finding has a fundamental impact on dentistry: because a lot of studies used these approaches as a reference method for different tasks, all these investigations and the conclusions drawn from them have to be carefully re-evaluated, especially statements concerning the locations of hinge axes (Hugger et al. 2001; Nagy et al. 2002; Bias and Kordass 2009; Morneburg and Pröschel 2011; Hugger and Kordass 2017; Okeson 2019).

The calculations and simulations here were mainly applied in a 2D environment. In general, for hinge axis determination this is not a very relevant restriction, because, if there are 3D measured points, they can be transformed into the sagittal plane.Further, the pantographic methods are by definition using the sagittal plane e.g. for recording the movement of the pin on a sheet or recording plane. The only prerequisite is that the sagittal plane or parallel planes have to be determined or defined beforehand. This involves an additional inaccuracy and therefore influences the values in Tab. 1. However, it is easy to show that the error using a plane, which deviates with an angle γ from the real sagittal plane, is proportional to the $\cos(\gamma)$. In case of a deviation angle γ of only a few degrees, the difference is very small and therefore negligible.

Additionally, there is to date an ongoing discussion about the relevance and applicability of the hinge axis concept. Firstly, there is no consensus at all, if the mandible performs a pure rotation during the first phase of mouth opening or last phase of mouth closing (Ahn et al. 2015; Ferrario et al. 1996; Gallo et al. 2008; Hellsing et al. 1995; Lindauer 1995; Nagy et al. 2002; Palla et al. 2003; Thieme et al. 2006; Torii 1989). If there would not be a pure rotation, it is debatable, which “rotation axis” instead is then to be determined (e.g. instantaneous axis, finite axis, kinematic axis) and how this does influence the further process (Chen and Katona 1999; Naeije and Hofmann 2003). Secondly, there are only few studies investigating the accuracy of such hinge axis determination devices (Bosman 1974; Hugger et al. 2001). The possible effect of this uncertainty on further steps for articulator programming and the measurement of other articulator parameters remains also to be investigated.

Conflict of interest statement:

The author declares that there is no conflict of interest for this study.

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Figure Legends:

Fig. 1 Left: Principle of the trace tracking method: A pin (P) fixed to a device (D_LJ), which is connected to the mandibular teeth, traces the movements on a frame (F), which itself is fixed to the head (or upper jaw) as a reference system (D_UJ). Right: In case of a pure rotational movement, the pin traces exact circular segments whose sizes depend on the distance to the center of rotation. The closer the position of the pin to the axis of rotation, the shorter the circular segment is. This method is also known as pantographic kinematic method and can also be implemented by electronic tracking devices.

Fig. 2: Temporomandibular joint with some parameters and variables used in the calculations or simulations respectively: Global coordinate system (X,Y) is fixed to the head and the local coordinate system (x,y) is fixed to the mandible. *O* is assumed to be the arbitrary reference point on the mandible, where angular and translational velocity is known, and *P* an arbitrary point fixed to the mandible. $\Delta\alpha$ is the rotation angle and *r* the distance of the incisal point from point *O.*

Fig. 3: Relevant feature points and distances in case of a uniform rotational and translational movement of the temporomandibular joint (as denoted in Tab. 1). The rotation is assumed around a reference point O with a constant angular velocity $\Delta\alpha/T$, simultaneously shifting this point O with constant velocities $\Delta x/T$ and $\Delta y/T$. The point with the least movement lies at position r_{x_0} and r_{y_0} relative to the reference point O with the respective distance d_{axis}.

Fig. 4: Results of the simulation with Matlab for a jaw movement (as shown in Fig. 3), where the condyle performs a combined uniform translational and rotational movement: 4a) with a translation of 1 mm in x- and 0.5 mm in y-direction (total 1.1 mm) and 4c) with a translation of 2 mm in x and 1 mm in y-direction (total 2.2 mm). The magnification of the respective traces around the finite center of rotation (FCR, corresponds approximately also to the point P_{Min}) are shown in 4b) and 4d), respectively. The maximum amplitudes (maximum length of the segments) of the respective traces were also calculated and plotted into the figures.

Mouth Opening $\Delta\alpha$			Translation	Position of Center P_{Min}			Path at P_{Min}
[mm]	°[deg]	[rad]	Δx [mm]	r_{x_0} [mm]	r_{y_0} [mm]	d_{axis} [mm]	Δl_{min} [mm]
15	9.6	0.166	**0.5**	0.25	3.00	**3.01**	0.0104
15	9.6	0.166	**1.0**	0.50	5.98	**6.00**	0.0208
15	9.6	0.166	**1.5**	0.75	8.97	**9.00**	0.0313
15	9.6	0.166	**2.0**	1.00	11.97	**12.01**	0.0417
10	6.4	0.111	**0.5**	0.25	4.49	**4.50**	0.0070
10	6.4	0.111	**1.0**	0.50	8.97	**8.98**	0.0139
10	6.4	0.111	**1.5**	0.75	13.48	**13.50**	0.0209
10	6.4	0.111	**2.0**	1.00	17.97	**18.00**	0.0278
5	3.2	0.056	**0.5**	0.25	9.00	**9.00**	0.0035
5	3.2	0.056	**1.0**	0.50	17.99	**18.00**	0.0070
5	3.2	0.056	**1.5**	0.75	26.98	**26.99**	0.0104
5	3.2	0.056	**2.0**	1.00	35.97	**35.98**	0.0139

Tab. 1:

Influence of different mouth opening and translation values on the displacement of the position of the (finite) center of rotation P_{Min}. The corresponding parameters are shown in Fig. 2 and some examples are displayed in Fig. 3 and 4. The rotation is assumed around a reference point O with a constant angular velocity $\Delta\alpha/T$, simultaneously shifting this point O with constant velocities $\Delta x/T$ and $\Delta y/T$. The point with the least movement lies at position r_{x_0} and r_{y_0} relative to the reference point O with the respective distance d_{axis} (see Fig. 2). Δl_{min} is the maximum observable movement of this point P_{Min}.